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METEOROLOGICAL RADAR CALIBRATION

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I. INTRODUCTION

The use of radar for the remote quantitative measurement of rain rate provides a powerful tool for those concerned with the distribution and occurrence of rainfall in the atmosphere. A knowledge of the spatial and temporal distributions of rainfall is essential in such diverse disciplines as meteorology, hydrology, and millimeter wavelength communications. Radar has been extensively used for applications such as these since the development of meteorological radar techniques in the 1940's. Nevertheless, the use of this approach has been handicapped by a lack of adequate precision in the resulting quantitative rain rate data: for example, Harrold and Nicholass [1] report average uncertainties in such radar measurements ranging from 30 to 100%. These uncertainties are largely attributable to the difficulties encountered in performing an adequate calibration of the radar measurement system.

Traditionally two distinct techniques have been used for radar calibration. The first method utilizes a metallic sphere or other target having a known radar cross section to establish an absolute calibration. Unfortunately, this method is based upon the use of a point target and does not compensate for the effects of beam-filled targets such as those usually encountered in rain rate measurements. Furthermore, it is often quite difficult to place such a standard calibration target in the far field of the radar antenna and at a height sufficient to eliminate ground reflections. The second common method is to calibrate the radar observed rain rate against the rain rate measured on the ground below a given radar resolution cell. This approach obviously ignores differences resulting from both vertical and horizontal wind components since the two measurements to be compared are made at two different spatial locations. This difficulty is compounded by the fact that the radar measurement must be elevated in order to avoid ground reflections.

This report addresses the problem of radar calibration from several different directions. It is shown in the following that the integrated, range-corrected radar backscatter saturates in a manner analogous to that found in radiometric temperature measurements. Furthermore, it is shown that this saturation leads to an absolute calibration method for radars operating at attenuating wavelengths. The method for calibration of meteorological radars in terms of independent radiometric temperature measurements is also developed. This technique offers the advantage that the calibration is based upon an independent measurement made throughout the same spatial region as that viewed by the radar.

II. THE RADAR RANGE EQUATION AND ITS SOLUTION

The development below follows closely that originally presented by Hitschfeld and Borden [2]. Here it is assumed that a narrow beam, short pulse radar is viewing a region of rainfall which fills each radar resolution cell homogeneously. Then, if the power pattern of the radar antenna is assumed to be Gaussian, the following radar range equation results [3]:

$$(1) \qquad P_r = \frac{Q}{r^2} Z e^{-2A_{RD}} \text{ [watts]}$$

where P_r is the time average received power backscattered from a resolution cell located at a range of r meters from the radar. The radar calibration constant, Q, is

$$(2) \qquad Q = \frac{\pi^5 |k|^2 f^2 G \tau P_t}{2^{10} \cdot \ln 2 \cdot c} \text{ [watts/meter]}$$

where

$$|k|^2 = \left|\frac{n^2-1}{n^2+2}\right|^2 ,$$

(3)

n = the complex refractive index of liquid water

f = radar frequency [H_z]

G = maximum antenna gain

τ = radar pulse length [sec.]

P_t = peak transmitted power [watts]

c = velocity of propagation [m/sec].

The equivalent reflectivity, Z, has been defined in terms of the volumetric radar cross section, [m^2/m^3]:

$$(4) \qquad Z = \frac{c^4 \eta}{\pi^5 f^4 |k|^2} \quad [m^6/m^3].$$

A_{RD} is the one-way attenuation between the radar and the resolution cell being observed expressed in terms of specific attenuation, α_{RD}[nepers/m], as

(5) $$A_{RD} = \int_0^r \alpha_{RD} dr \text{ [nepers].}$$

Several additional assumptions will be incorporated in the following development. These include the assumption that both the specific attenuation, α, and the reflectivity, Z, may be represented at any frequency by a power law dependence on the rain rate, R [mm/hr], i.e.,

(6) $$\alpha_{\substack{RD\\RM}} = a_{\substack{RD\\RM}} R^{b_{\substack{RD\\RM}}}.$$

and

(7) $$Z = a_Z R^{b_Z}$$

where the subscripts RD and RM indicate that the subscripted quantity is to be evaluated at either the radar or radiometer frequency, respectively. Calculations based on Mie theory for various drop size distributions [4,5] indicate that this is a reasonable assumption even for frequencies where the drop sizes are resonant; this is demonstrated by Figures 1 and 2 which show regression power low fits to Mie calculations at frequencies from 1 to 500 GHz for Marshall-Palmer drop size distributions at 0°C. The a and b constants in Equations (6) and (7) are, of course, frequency dependent and have been tabulated for extensive frequency ranges in Reference 4 for the case of specific attenuation and in Reference 5 for both specific attenuation and reflectivity.

It will also be assumed that attenuation and radiometric temperature, T_s [°K], are related by

(8) $$A_\infty = \ln\left(\frac{T_m}{T_m - T_s}\right)$$

where T_m [°K] is the mean absorption temperature of the medium and the subscript ∞ indicates that the attenuation is associated with a path extending to a resolution cell located at infinity [6]. This theoretical relationship is strictly valid for media where scattering is negligible; however, it has been demonstrated empirically that it is quite adequate for the rainfall case at frequencies at least up to 15 GHz [7,8]. An example of this agreement is shown in Figure 3 for the case of simultaneous 15 GHz radiometric temperature measurements and attenuation measurements using the ATS-5 synchronous satellite beacon.

Now, proceeding with the solution of the radar range equation, the rain rate, R, is eliminated from Equations (6) and (7) yielding

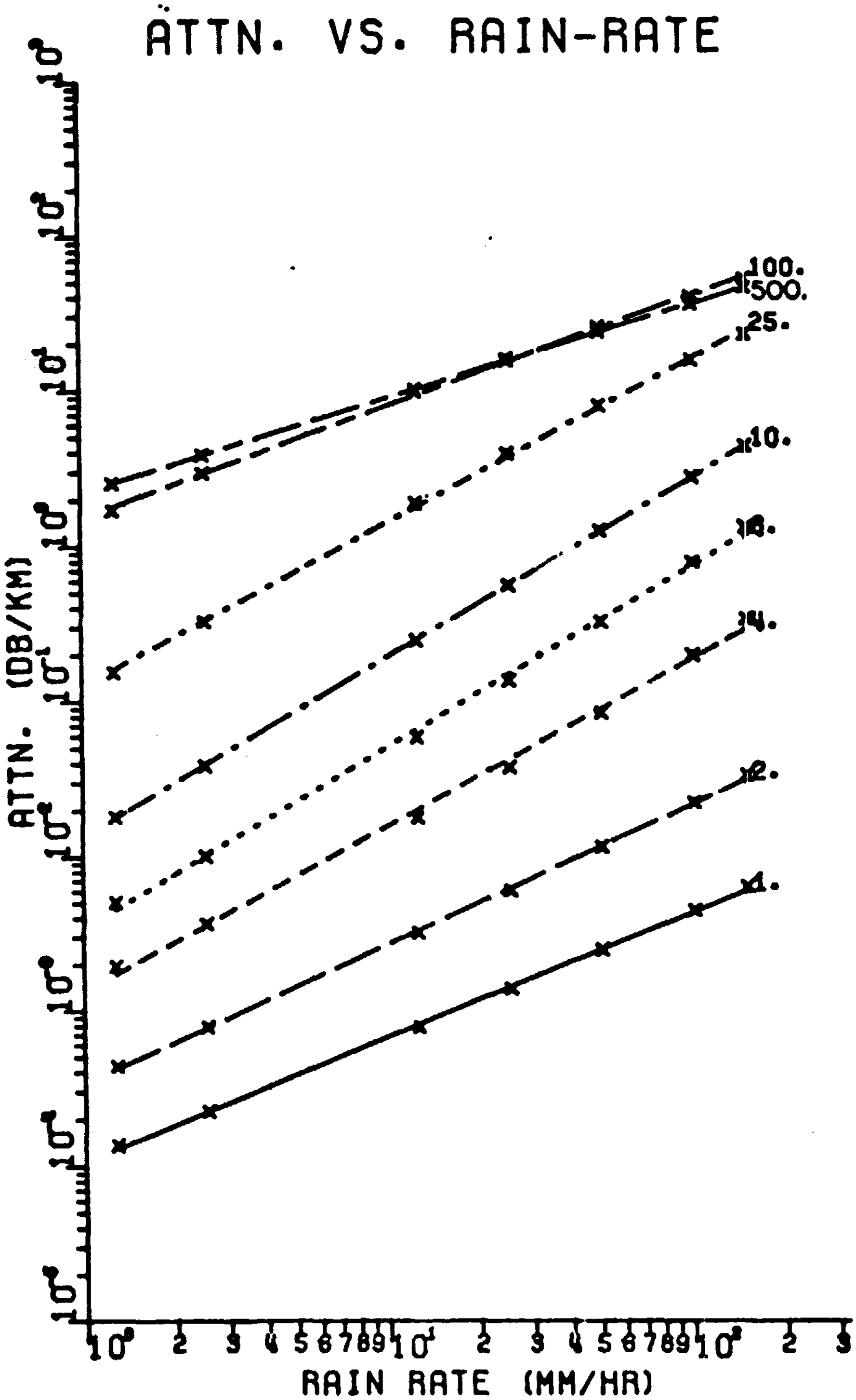

Figure 1. Specific attenuation, α , vs. rain rate at several frequencies.

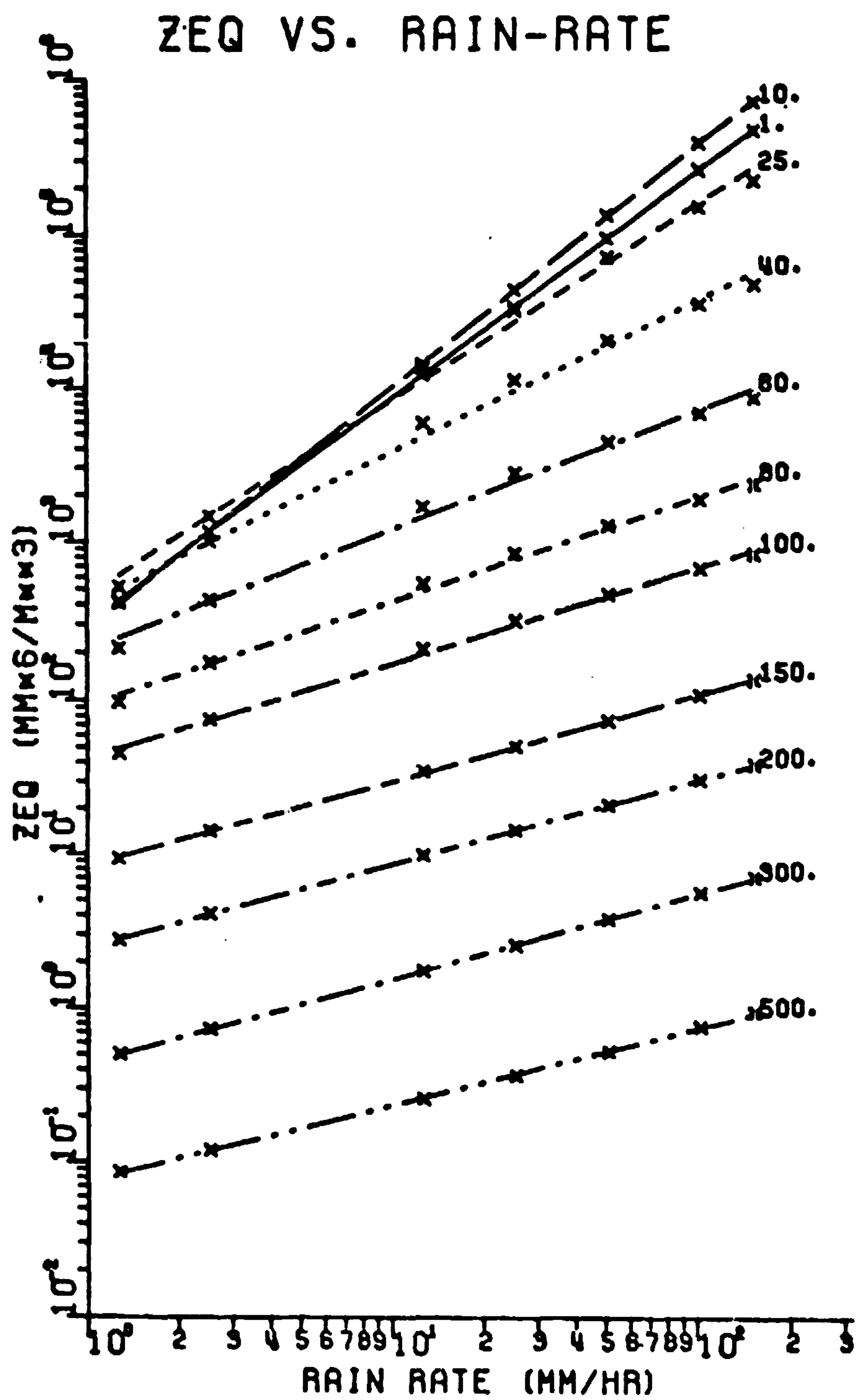

Figure 2. Equivalent reflectivity, Z_{eq}, vs. rain rate at several frequencies.

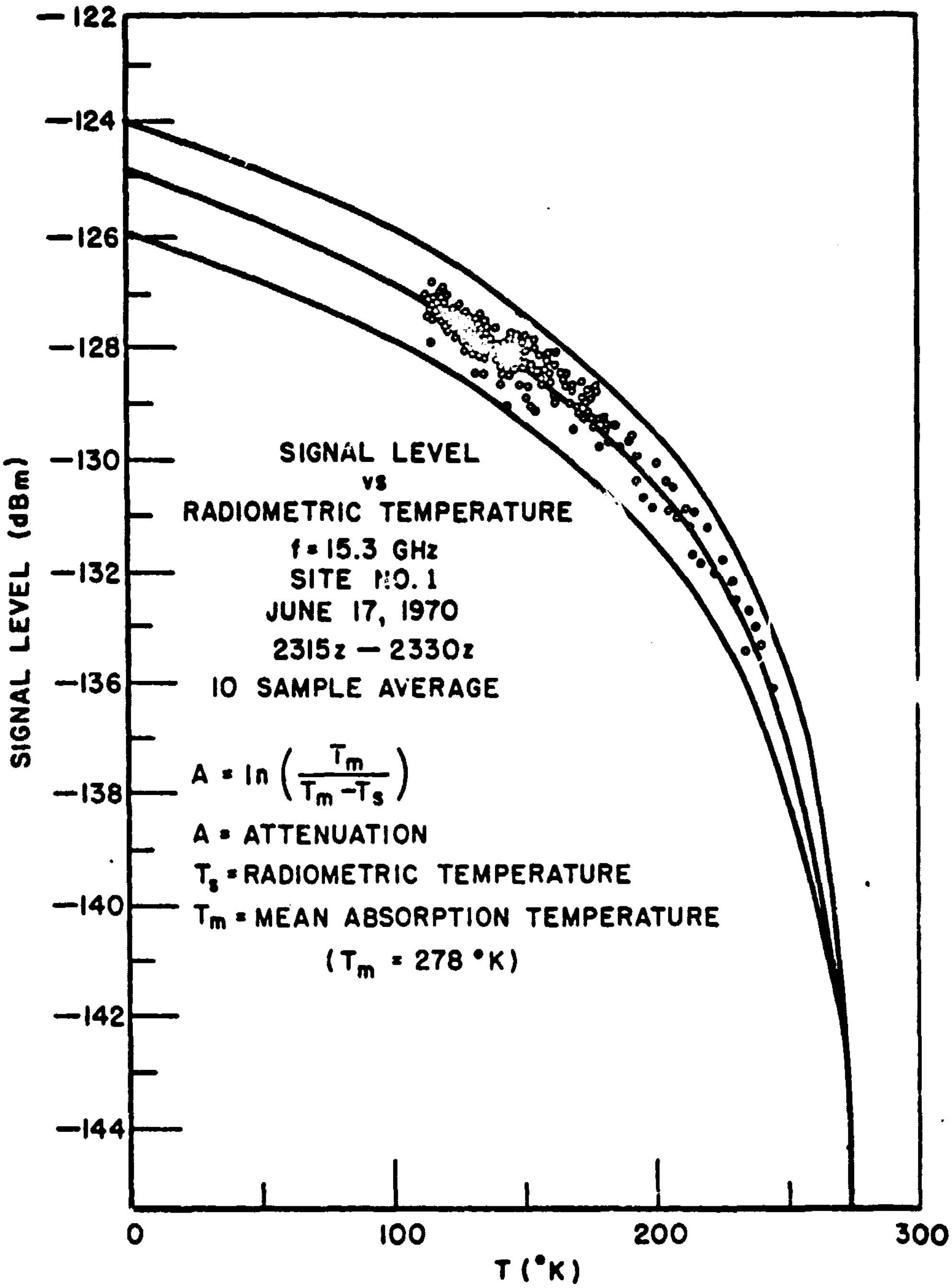

Figure 3. A comparison between measured attenuation and measured radiometric temperature at 15 GHz [8].

$$\alpha_{RD} = a_{RD}\left(\frac{Z}{a_Z}\right)^{\frac{b_{RD}}{b_Z}} . \quad (9)$$

Combining Equations (1), (5), and (9), the radar range equation is expressed in terms of equivalent reflectivity:

$$r^2 P_r = QZe^{-2\left[\frac{a_{RD}}{a_Z{}^{b_{RD}/b_Z}} \int_0^r Z^{b_{RD}/b_Z}\, dr.\right]} \quad (10)$$

Making the substitutions

$$g = 2\,\frac{a_{RD}}{a_Z{}^{b_{RD}/b_Z}} \quad (11)$$

and

$$y = Z^{-b_{RD}/b_Z} , \quad (12)$$

for simplicity, yields

$$r^2 P_r = Q\, y^{-b_Z/b_{RD}}\, e^{-g\int_0^r \frac{1}{y}\, dr}. \quad (13)$$

Now, the logarithm is taken of Equation (13); the result is then differentiated with respect to r and rearranged:

$$\frac{dy}{dr} + \frac{b_{RD}}{b_Z} \cdot \frac{d[\ell n(r^2 P_r)]}{dr}\; y = -\frac{b_{RD}}{b_Z}\; g. \quad (14)$$

The solution of Equation (14) is

$$y = \frac{c - \frac{b_{RD}}{b_Z} \cdot g\int_0^r (r^2 P_r)^{\frac{b_{RD}}{b_Z}}\, dr}{(r^2 P_r)^{b_{RD}/b_Z}} \quad (15)$$

where c is an unknown integration constant. Substituting for g and y, the solution,

$$Z = \frac{r^2 P_r}{\left[c - 2\,\frac{b_{RD}}{b_Z}\,\frac{a_{RD}}{a_Z{}^{b_{RD}/b_Z}} \int_0^r (r^2 P_r)^{b_{RD}/b_Z} dr\right]^{b_Z/b_{RD}}} \quad (16)$$

is obtained.

The integration constant, C, may be evaluated by considering the simple hypothetical reflectivity distribution shown in Figure 4.

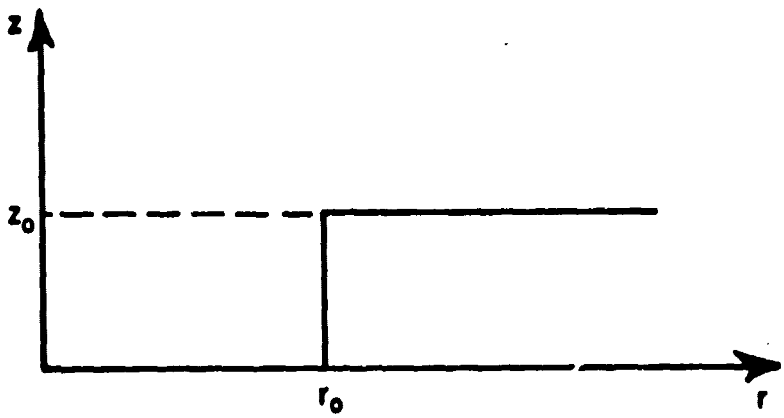

Figure 4. Reflectivity distribution used to evaluate the unknown constant of integration, c.

Since the attenuation over the interval from r=0 to r_o is zero, Equation (1) becomes

(17) $$P_r = \frac{Q}{r_o^2} Z_o$$

when evaluated for $r=r_o$. Similarly, $P_r=0$ for $0<r<r_o$ and, therefore, Equation (16) becomes

(18) $$Z_o = \frac{r_o^2 P_r}{c^{b_z/b_{RD}}}$$

when evaluated for $r=r_o$. Consequently,

(19) $$c = Q^{b_{RD}/b_z} .$$

Substituting Equation (19) into Equation (16), the following compact form of the solution of the range equation is obtained

(20) $$Z = \frac{r^2 P_r}{Q(1-I_r)^{b_z/b_{RD}}}$$

where

$$(21) \qquad I_r = \frac{2a_{RD}b_{RD}}{b_z(a_zQ)^{b_{RD}/b_z}} \int_0^r (r^2P_r)^{b_{RD}/b_z} dr \, .$$

III. THE RADAR SATURATION FACTOR

The quantity I_r has a number of interesting properties. For example, comparing Equations (1) and (20), it is noted that

$$(22) \qquad A_{RD} = \ell n \frac{1}{(1-I_r)^{b_z/2b_{RD}}} \, .$$

Thus, the quantity I_r is intimately related with the path attenuation. Furthermore, equating Equations (8) and (22) for a resolution cell located at infinity, it is found that

$$(23) \qquad 1 - \frac{T_s}{T_m} = (1-I_\infty)^{b_z/2b_{RD}}$$

where, again, the subscript ∞ indicates that the integration is to be performed over the entire range from $r=0$ to ∞. Here it is seen that I_∞ is closely related to the ratio of the radiometric sky temperature, T_s, to the mean absorption temperature, T_m. Since the ratio T_s/T_m is a measure of the degree of saturation associated with the radiometric temperature measurement, it is reasonable to associate that interpretation with the radar case as well. Thus, the quantity I_r will be referred to as the radar saturation factor.

The radar saturation factor, I_r, is unitless, ranges from zero to one, and increases monotonically with range. Examining Equation (22) it will be noted that, as I_r approaches unity, the path attenuation becomes extremely large.

Examining Equation (21), it is noted that the calculation of I_r involves a very simple range correction and integration of the received backscattered radar power. Thus, the real time calculation of I_r provides a simple means of determining, in real time, whether a radar has become "blind" due to range attenuation and, furthermore, at what range the "blindness" occurs.

Now consider an extended region of uniform rain rate and, hence, uniform reflectivity as depicted in Figure 5. Examining Equation (20) it is clear that the quantity

$$\frac{r^2 P_r}{(1-I_r)^{b_Z/b_{RD}}} = QZ \tag{24}$$

is a constant independent of range since both Q and Z are independent of range. Furthermore, since the attenuation will increase without bound as r increases, the range corrected received power, $r^2 P_r$, must approach zero (see Equation (1)). Therefore, the radar saturation factor must approach unity as the range increases.

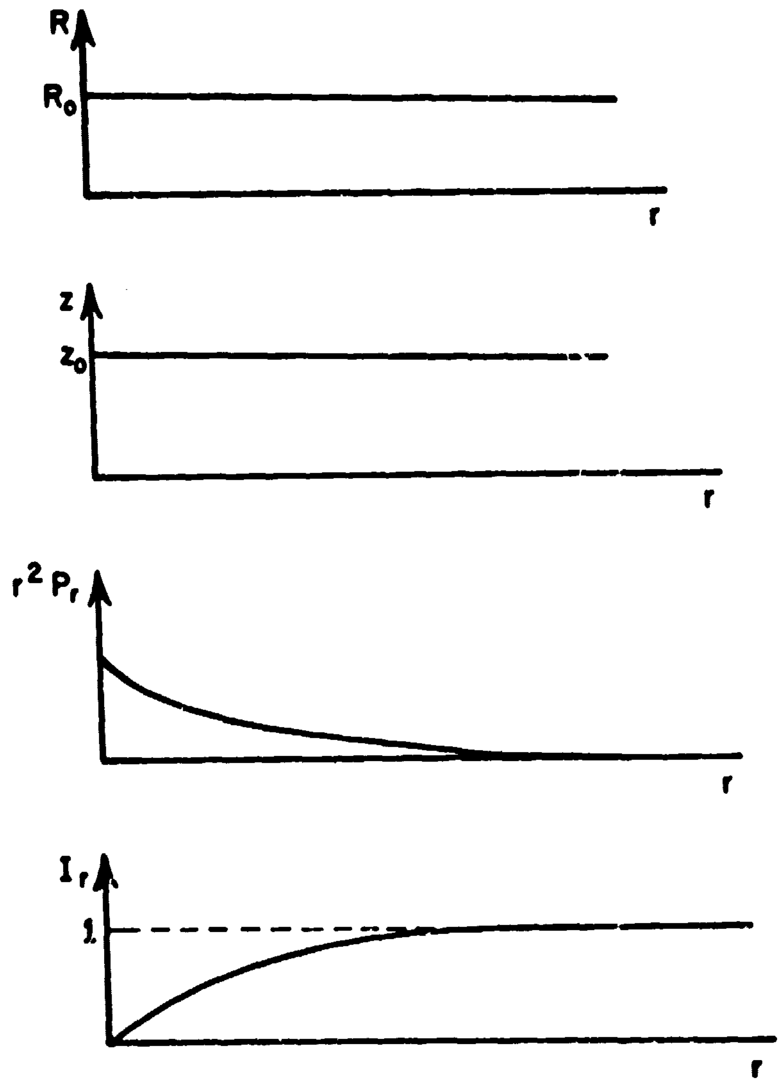

Figure 5. The case of extended uniform rain rate.

Similar reasoning leads to the same conclusion even for the case of an inhomogeneous rain rate distribution if either the rain rate or extent is sufficiently large, i.e.,

$$I_r \to 1 \tag{25}$$

for any case where the path attenuation becomes sufficiently large. Substituting Equation (21) into (25) and rearranging yields

$$\frac{1}{a_Z}\left[\frac{2a_{RD}b_{RD}}{b_Z}\int_0^\infty (r^2 P_r)^{b_{RD}/b_Z} dr\right]^{b_Z/b_{RD}} \to Q \tag{26}$$

for sufficiently high or extended rain rates. The left side of Equation (26) has an absolute bound which is simply the calibration constant of the radar. Thus, if the left side of Equation (26) is evaluated and the maximum value found for experimental data obtained using an attenuating wavelength radar viewing a sufficiently intense storm, this value establishes an <u>absolute</u> radar calibration constant <u>independent</u> of any other measurements.

IV. RADIOMETRIC TEMPERATURE

Since the reflectivity, Z, can be found as a function of range, Equation (20), the rain rate, R, is also known, Equation (7):

$$R = \left(\frac{r^2 P_r}{a_Z Q}\right)^{1/b_Z} \frac{1}{(1-I_r)^{1/b_{RD}}} \quad . \tag{27}$$

Similarly, using Equation (6) the specific attenuation can be found at any other frequency. In particular, let the specific attenuation be evaluated for some radiometer frequency other than the radar frequency; then,

$$\alpha_{RM} = a_{RM}\left(\frac{r^2 P_r}{a_Z Q}\right)^{b_{RM}/b_Z} \frac{1}{(1-I_r)^{b_{RM}/b_{RD}}} \tag{28}$$

and the path attenuation at the radiometer frequency is

$$A_{RM_\infty} = \frac{a_{RM}}{(a_Z Q)^{b_{RM}/b_Z}} \int_0^\infty \frac{(r^2 P_r)^{b_{RM}/b_Z}}{(1-I_r)^{b_{RM}/b_{RD}}}\, dr. \tag{29}$$

Combining Equations (8) and (29) yields

$$(30) \qquad T_s = T_m\left\{1-\exp\left[-\frac{a_{RM}}{(a_zQ)^{b_{RM}/b_z}}\int_o^\infty \frac{(r^2P_r)^{b_{RM}/b_z}}{(1-I_r)^{b_{RM}/b_{RD}}}\,dr\right]\right\}$$

so that the radiometric sky temperature can be obtained at any frequency directly from the radar data.

V. RADAR CALIBRATION

If Equation (30) is rearranged, the radar calibration constant, Q, can be expressed in terms of the backscattered radar power and the radiometric temperature as:

$$(31) \qquad Q = \frac{1}{a_z}\left[\frac{a_{RM}\int_o^\infty \frac{(r^2P_r)^{b_{RM}/b_z}}{(1-I_r)^{b_{RM}/b_{RD}}}\,dr}{\ln\left(\frac{T_m}{T_m-T_s}\right)}\right]^{b_z/b_{RM}}$$

The right hand side of this expression is implicitly dependent upon Q because of the presence of I_r. Nevertheless, if a reasonable estimate of Q can be made, this expression can be used to solve for Q in a stable, iterative manner. The estimate of Q should be used to calculate the radar saturation factor, I_r, and then the improved estimate of Q is calculated using Equation (31). Thus, a bore-sight radiometer operating at a frequency either the same as or different than the radar frequency can be used to calibrate the radar on an instantaneous basis.

Obviously, if the radar is operating at a non-attenuating wavelength or if the rainfall observed is very light, the radar saturation may be very small or even negligible; in this case, neglecting I_r in Equation (31) eliminates the need for an iterative solution of this equation.

Conversely, if very intense rain is observed at an attenuating wavelength, the radar saturation factor, I_r, will approach unity for large r; in this case the integrand in Equation (31) will become indeterminate and, thus, may generate significant errors. However, this is precisely the case where the bound given by Equation (26) is applicable; thus, the radiometric temperature measurement and the use of Equation (31) are not required.

VI. SUMMARY

Using the Hitschfeld-Borden approach, the radar reflectivity, Z, at a given range has been obtained in terms of the received radar power, the radar calibration constant, and the parameters of the power law relations between specific attenuation, reflectivity, and rain rate. This solution of the radar range equation contains a factor, I_r, referred to as the radar saturation factor, which is dependent upon the integral of the range corrected received radar power. This radar saturation factor, in turn, is shown to exhibit many interesting characteristics; for example, this factor is very simply related to the path attenuation and the radiometric temperature. Furthermore, this factor is shown to saturate, i.e., approach a known bound, in a manner analogous to that in which radiometric temperature saturates. Finally, these properties are used to develop two alternative techniques useful for the determination of the radar calibration constant. The first technique, applicable to radars operating at attenuating wavelengths, permits a direct, absolute calibration of the radar in terms of the measured radar data itself. The second technique, employing independent radiometric temperature measurements, permits continuous radar calibration during data acquisition.

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